Project Icarus: A Design and Development of Rocket Nozzles for High-Pressure Atmospheres

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Background: Rocket nozzles, which determine the speed and shape of rocket thrust, undergo overexpansion in high-pressure atmospheres. Using hand calculations, Rocket Propulsion Analysis (RPA), CAD software, and computational fluid dynamics (CFD), we designed and compared three sets of near-optimal bell and conical rocket nozzles for their relative efficiencies in the high-pressure atmospheres of Earth, Titan, and Saturn. **Methods:** Thrust and isentropic flow equations, along with the Chemical Equilibrium with Applications interface (CEARUN) were used to size six rocket engines and nozzles, with engines being held constant according to the studied celestial body. Blank nozzle contours, estimated efficiencies, and 2D geometries were then generated and meshed to ensure accuracy in CFD simulations. The exit velocities of these fluid simulations, which directly correlate with nozzle efficiency, were compared between the conical and bell nozzles using analytical and graphic methods (data tables and value contours, respectively). **Results:** Compared to conical rocket nozzles, bell nozzles had greater exit velocities in Earthen and Saturnian atmospheric conditions. When simulating Titan's atmosphere, the conical rocket nozzle had a higher exit velocity than that of the bell nozzle. The peak velocities of the conical nozzles were equal to their exit velocities, while the bell nozzles' peak velocities differed from their exit velocities in the atmospheric conditions of Earth and Titan. **Conclusion:** This study and design project showed the effectiveness of thrust and isentropic flow equations in modeling accurate rocket nozzles for high-pressure atmospheres, along with the relative efficiencies of our bell and conical designs in Earth-, Titan-, and Saturn-based CFD simulations. Due to the presence of thrust losses near our bell-nozzle exit areas, further experiments will include better-optimized bell-nozzle sizing parameters.

Keywords: Bell rocket nozzle · Conical rocket nozzle · Rocket nozzle efficiency · Ambient pressure · Overexpansion · Expansion ratio · Hand calculations · Isentropic flow · Computational fluid dynamics (CFD) · Exit velocity · Thrust loss

INTRODUCTION:

Nozzles are a crucial aspect of the modern rocket, characterized by their ability to control the shape and velocity of thrust, which is the outward expulsion of gases burned within the rocket's combustion chamber, allowing for flight (Khare & Saha, 2021). Large-scale space exploration missions are heavily reliant on the design of rocket nozzles, as slight alterations in nozzle-sizing parameters give way for significant variations in fuel and resource efficiency; thrust loss must be mitigated through careful optimization of these dimensions, taking into account the ambient pressure of the operating environment (Sutton & Biblarz, 2010). Past studies and models have made use of this relationship, with numerous optimal nozzles having been created for the Earthen and lunar atmospheres (Khare & Saha, 2021). As the field of extraterrestrial science advances, however, it becomes difficult to utilize Earth-adapted designs in areas of greater pressure, such as Saturn and its largest moon, Titan.

In the case of high-pressure celestial bodies, overexpansion of thrust is likely to occur due to the exhaust-plume pressure being lower than that of the surrounding atmosphere (Kurbel, 2017). This causes losses in efficiency, as the full extent of the nozzle wall is not used to increase the gas exit velocity. Furthermore, excess fuel expenditure and unequal heating of the nozzle are more prevalent with this suboptimal expansion type, leading to higher mission costs and possible thermal damage (Kurbel, 2017).

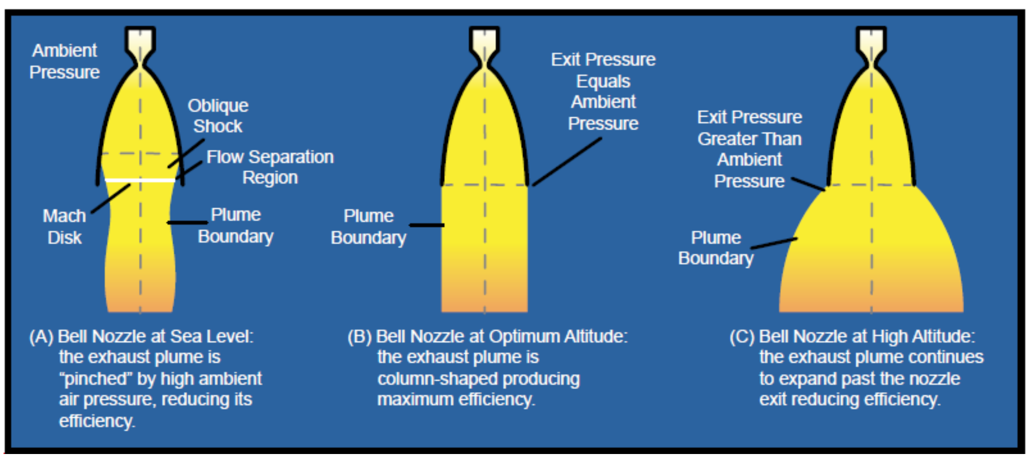

Figure 1. Bell rocket nozzle contours displaying (A) overexpansion, (B) optimal expansion, and (C) underexpansion (from Kurbel, B. 2017, April 18, Peak of Flight: Apogee Rockets)

In order to counteract these atmospheric effects, the area expansion ratio from the nozzle's throat to exit must be determined with respect to the chosen fuel, oxidizer, and ambient pressure (Sutton & Biblarz, 2010). An ideal rocket nozzle will have an exit pressure equal to that of the surrounding atmosphere, allowing for a rectangular spread of thrust (Figure 1, B). The required length of a nozzle increases with the area expanded, meaning that material usage and weight can be reduced through selecting a design that minimizes length while retaining an equal expansion ratio. Research shows that bell nozzles are better optimized in this regard than their conical counterparts, as they expand an equal amount despite their smaller size; this phenomenon is attributed to their curved nature, which consists of converging and diverging regions with large angles that taper towards the exit (Figure 2) (Arrington et al., 1996).

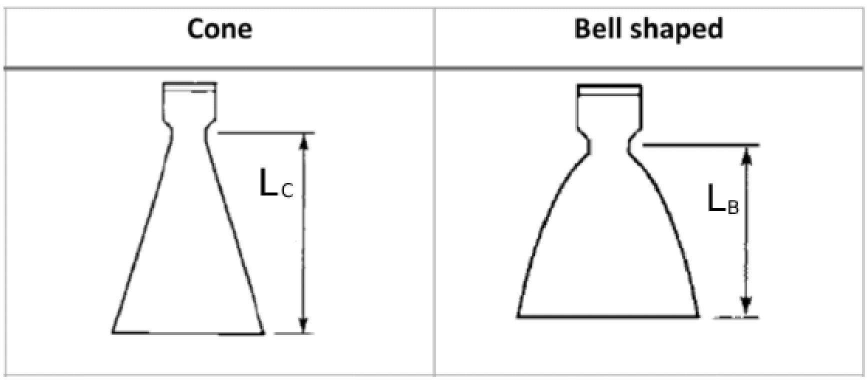

Figure 2. Cone and bell-shaped rocket nozzles with equal expansion ratios, but varying lengths, where $L_C > L_B$ (from Baidya, R. et al, 2018, Ramjet Nozzle Analysis for Transport Aircraft Configuration for Sustained Hypersonic Flight)

While the conical nozzle lacks critical weight optimization, this model still finds its use within lower-scale missions due to its simplicity in development and manufacturing. Moreover, the benefits provided by a bell-shaped design are accompanied by their own set of challenges, namely the increased presence of oblique shock waves within the nozzle, which cause flow separation and decrease practical efficiency (Hunter, 1998). As such, when developing a rocket nozzle for future space missions, it is key that a variety of traditional models are considered, including both bell and conical forms. Possible areas of exploration include the aforementioned Saturn and Titan, both of which contain high-pressure atmospheres, similar to that of Earth.

Recent classifications of Titan's surface makeup, which include organic chemicals and liquid water, prove its promise in the search for extraterrestrial life, among other major breakthroughs (Poggiali et al., 2024). Despite this, however, only one spacecraft—the Huygens Probe—has ever been sent to this lunar body. The largely untapped potential of Titan has recently gained significant recognition by the scientific community, meaning that on-site exploration using rocket-powered spacecraft is likely to take place in the near future (Poggiali et al., 2024).

Titan's host planet, Saturn, faces a similar situation in which rocket-based research has not yet been conducted in the planet's lower-level airspace; this makes the two celestial bodies apt for novel, experimental design. Earth also finds its place as a meaningful control group due to its high-pressure atmosphere being akin to that of the former two bodies.

Main Objectives:

To design three pairs of bell-shaped and conical rocket nozzles suited for the high-pressure atmospheres of Titan, Saturn, and Earth.

To determine whether bell or conical nozzle designs are more optimal to minimize thrust loss in the studied bodies' atmospheric conditions, proving or denying the hypothesis that parabolic nozzles use thrust more efficiently.

METHODOLOGY:

Safety

Due to the purely computational nature of Project Icarus, formal safety measures were deemed unnecessary. No risks were identified within the methodology, and all procedures took place in protected virtual environments.

Design Variables

Symbol	Description
g	Acceleration due to gravity (m/s)
G	Gravitational constant 6.67408×10^{-11} m^3 kg^{-1} s^{-2}
M	Mass (kg)
R	Radius (m)
a	Altitude (m)
F	Thrust (N)
ϵ, Ae/At	Throat-exit area expansion ratio
P	Pressure (bar)
γ	Specific heat ratio
θ	Angle (°)
Dc	Diameter of chamber (mm)
Dt	Diameter of throat (mm)
Lcyl	Length of cylinder (mm)
Lc	Length of converging region (mm)
L*	Characteristic length (mm)
R1	Initial radius of converging region (mm)
R2	Final radius of converging region (mm)
b	Break angle (°)
Ac/At	Chamber-throat area expansion ratio
Rn	Radius of throat contour (mm)
Tn	Throat angle (°)
Te	Exit angle (°)
De	Diameter of exit (mm)
Le	Length of expanding region (mm)

Table 1. Constants and variables utilized and referenced, in order of appearance.

Prior to the development of our nozzle contours, the values for their respective rocket engines were found through numerical analyses. These consisted of fuel- and pressure-based calculations aided by the Chemical Equilibrium with Applications interface (CEARUN). The Merlin 1-D engine was chosen as a basis due to its usage in the Falcon Heavy Rocket, which is at the forefront of present-day space exploration missions; this includes the launch of NASA's Titan-exploring Dragonfly drone in 2028 (Foust, 2024; Wevolver, n.d.). Subsequently, RP-1—used by SpaceX's Merlin engine line—was decided as the fuel, with liquid oxygen (LOX) serving as its oxidizer (Wevolver, n.d.). Critical, averaged attributes of the Merlin 1-D engine and its combustion sources are listed below (Table 1).

Thrust at sea level on Earth (kN)	Chamber pressure (bar)	Propellant	Oxidizer/Fuel Ratio
954	108	LOX / RP-1	2.27

Table 2. Relevant characteristics of the Merlin 1-D engine and its combustion sources (modified from Wevolver, Merlin engine (Merlin-1D) - Falcon 9 & Falcon Heavy, n.d.)

Using these constraints, the desired thrust was determined through careful consideration of the rockets' operational altitudes, which were chosen to be sea-level on Earth and Titan, and approximately 225 kilometers below-sea-level on Saturn; the following formula was used to find the acceleration due to gravity at this altitude:

$$g = \frac{GM}{(R-a)^2} \tag{1}$$

The acceleration due to gravity at this altitude on Saturn was thus determined, along with those of Titan and Earth. The latter two values were found through empirical data rather than rigorous calculation. Similarly, past research shows the Merlin 1-D engine's thrust performance at sea level on Earth, allowing the following modified thrust equation to be used for the two remaining celestial bodies (Wevolver, n.d.):

$$F = F_E + (g - g_E) \tag{2}$$

In combination with atmospheric pressure, the ratio of specific heats and characteristic velocities of the RP-1/LOX compound were then determined per celestial body within CEARUN. A table-summary of the key attributes of Earth, Titan, and Saturn is shown below (Table 2), along with relevant, averaged values for each nozzle found through CEARUN (Table 4).

	Altitude (m)	Acceleration due to gravity (m/s²)	Atmospheric Pressure (bar)	Target Thrust (kN)
Earth	0	9.807	1.01325	854
Titan	0	1.352	1.5	845.545
Saturn	-225,308	10.365	10	854.558

Table 3. Relevant characteristics of Earth, Titan, and Saturn.

	Ratio of specific heats	Characteristic velocity (m/s)
Earth	1.12	1729.5
Titan	1.12	1737.7
Saturn	1.14	1775.3

Table 4. Relevant results from CEARUN for Merlin-based engine and fuel on Earth, Titan, and Saturn.

Using the previously discovered values, the area expansion ratios ϵ for each celestial body's rocket nozzles were found through the following equation:

$$\epsilon = \left(\left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}} \cdot \left(\frac{(\gamma+1)}{2}\right)^{\frac{1}{\gamma-1}} \cdot \sqrt{\frac{\gamma+1}{\gamma-1}\left(1-\left(\frac{P}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}}\right)} \right)^{-1} \quad (3)$$

Where P_1 is the chamber pressure, and γ is the ratio of specific heats of RP-1/LOX, as previously determined per celestial body (Table 2).

These values, when used as input for the Rocket Propulsion Analysis (RPA) software, allowed six different contours to be generated, with each celestial body having its own bell and conical rocket nozzle model. Half-angles for the conical designs were held constant at 15° to maintain stability and efficiency, while the initial and final angles for the bell rocket nozzles were determined using the initial and final angle *vs*. expansion ratio function discovered by Gadicharla V.R. Rao (Figure 3) (Sutton & Biblarz, 2010).

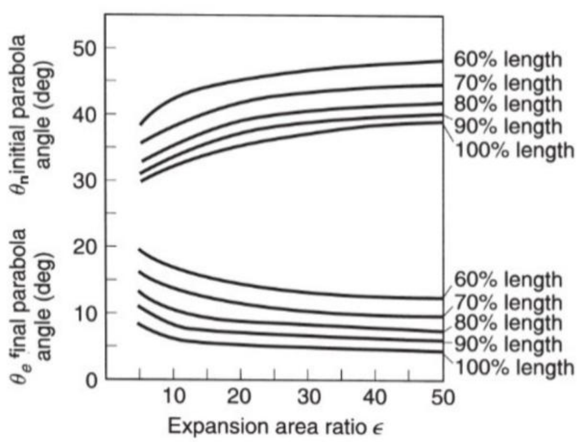

Figure 3. Initial angle θ_n and exit angle θ_e *vs*. expansion area ratio ϵ function for bell rocket nozzles, as found by G. V. R. Rao (from Sutton, G. P., & Biblarz, O. (2010). Rocket Propulsion Elements (8th ed.). John Wiley & Sons)

These results were saved in the Drawing Exchange Format (dxf) as half-contours and brought into Autodesk Fusion, where the software's sketch and surface tools were used to outline and generate the 2D geometry of each rocket nozzle. After exporting these designs as Standard ACIS Text (sat) files, they were imported into Ansys Workbench under a new Fluid Flow (Fluent) project.

In the Geometry menu of Workbench, 2D analyses were conducted using the DesignModeler software; the dimensions of each nozzle's combustion chamber, converging region, and diverging region were precisely defined using the line tool. These areas were promptly given workable surfaces using the Face Split tool, preparing each geometry for meshing within Ansys Mechanical (Patil et al., 2024).

The Face Mesh tool was applied after generating a default mesh for each rocket nozzle, significantly increasing the eventual accuracy of the computational fluid dynamics (CFD) simulation (Patil et al., 2024). Furthermore, Edge Meshing along each nozzle's walls allowed for heightened precision in areas more likely to endure thrust losses, meaning that the results would more clearly display the possible faults of each nozzle design. A sample mesh of the bell rocket nozzle for Titan can be seen in Figure 4.

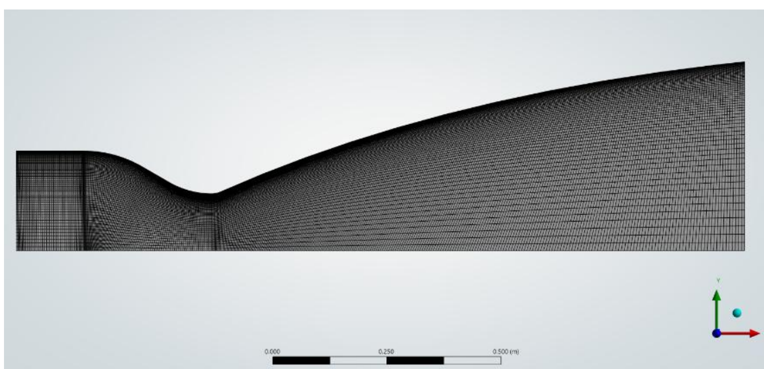

Figure 4. Completed mesh of our Titan-based bell rocket nozzle in Ansys Mechanical, where darker, finely meshed areas are subject to greater precision during CFD simulations.

Each design was then prepared for fluid flow simulation through the Setup menu, where the following settings were applied prior to launching the solver:

General	Solver Type: Density-based 2D Space: Axisymmetric
Models	Energy equation: On
	Viscous model: k-epsilon, standard
Materials	Fluid: air Density: ideal gas Viscosity: sutherland

Table 6. Setup parameters held constant among all six nozzles being designed and tested (adapted from Jayaprakash, P., Dhinarakaran, D., & Das, D., 2022, Design and analysis of a rocket C-D nozzle. International Journal of Health Sciences, 3545–3559)

Each nozzle also had average boundary conditions for its inlet and outlet regions, which are specified in the following table:

	Inlet Gauge Total Pressure (Pa)	Inlet Initial Gauge Pressure (Pa)	Outlet Gauge Pressure (Pa)	Outlet Backflow Total Temperature (K)
Earth	10901325	10900325	202325	846.0504
Titan	10950000	10949000	299300	874.0421
Saturn	11800000	11799000	2021300	1030.4282

Table 7. Average boundary conditions for each celestial body, with conical and bell designs having the same parameters per body. Chamber temperatures were held constant.

<u>RESULTS:</u>

Following the solving of each nozzle simulation, relevant outputs were displayed in the form of 2D contours. The variables studied include fluid pressure and velocity, which are inversely related, along with temperature (Patil et al., 2024). As shown in Figures 5–7, results varied significantly among the three celestial bodies, with Saturn's nozzle designs being particularly different from those of Earth and Titan.

Deeper analyses of the rocket nozzles' respective exit velocities revealed the relative efficiencies of our bell and conical designs. In the atmospheres of Earth and Saturn, the bell nozzles achieved greater thrust given equivalent inlet conditions to their conical counterparts. Conversely, the latter model had superior performance in Titan's atmosphere, signified by the conical nozzle's marginally greater exit velocity when simulated in the moon's surface conditions.

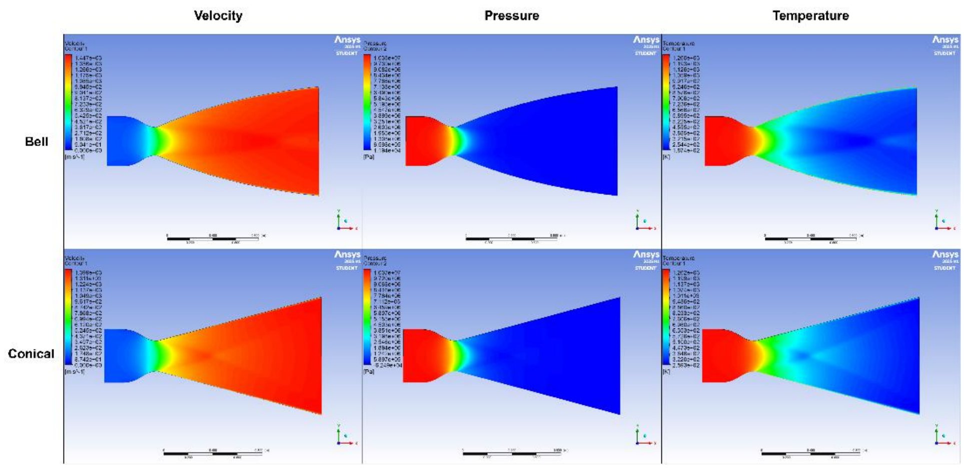

Figure 5. The Earth-adapted rocket nozzles' velocity, pressure, and temperature contours.

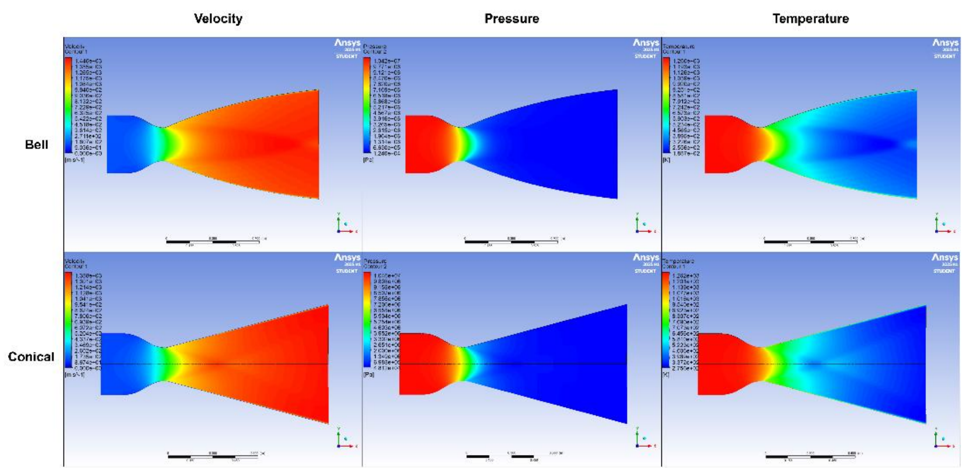

Figure 6. The Titan-adapted rocket nozzles' velocity, pressure, and temperature contours.

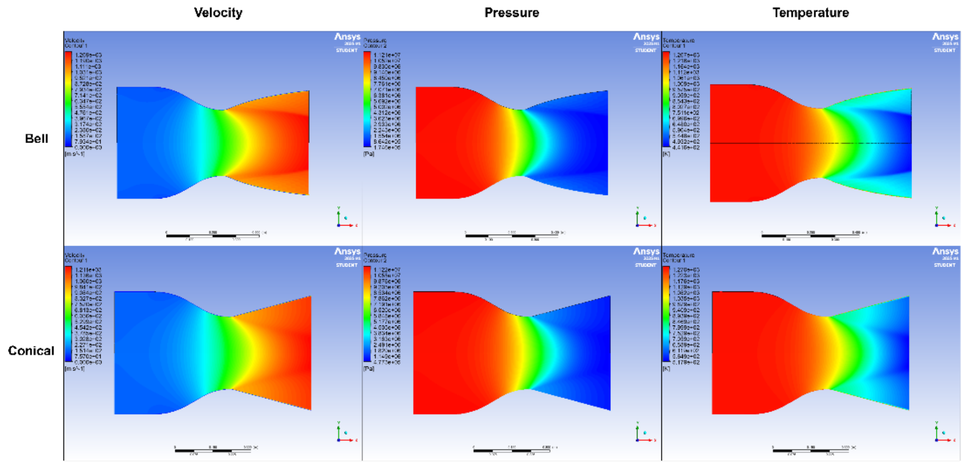

Figure 7. The Saturn-adapted rocket nozzles' velocity, pressure, and temperature contours.

DISCUSSION AND CONCLUSIONS

The expected relative efficiencies of our designs, made prior to experimentation, were compared with the results found through CFD simulations. Our hypothesis held true for two of the three studied bodies, namely Saturn and Earth, due to the bell nozzles' higher performance in their atmospheric conditions. Thrust losses were found to be present in the bell-nozzle exit regions of the Earth-based and Titan-based models (Figures 5 and 6), pinpointing the source of our adverse findings. Further analysis outside the scope of this project must be conducted to determine the exact causes of these losses in efficiency, though they can most likely be attributed to boundary layer separation, as was found in studies conducted at the NASA Langley Research Center (Hunter, 1998).

The relative exit velocities of the Earth- and Saturn-adapted rocket nozzles complied with past research, which often found that bell designs garnered higher thrust efficiencies than conical models. This was the case in a 1996 study on physical bell and conical rocket nozzles, which concluded that parabolic-contoured designs were far superior to their linear, conical counterparts in terms of thrust efficiency (Arrington et al., 1996).

Through mathematical and computational analysis, our research findings supported the widely held belief that bell rocket nozzles are more efficient than conical nozzles, while also providing novel models for potential use in future missions. As six rocket nozzles were able to be designed and simulated in their respective high-pressure environments, with both support and denial of the original hypothesis being found, the experiment was determined to be successful.

All main objectives were completed with near-optimal rocket nozzle designs, giving way for future testing, and possibly physical reproductions, of the models generated throughout Project Icarus.

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